TECHNICAL NOTE

JILES-ATHERTON MAGNETICS MODEL

OVERVIEW

This note describes the basic equations behind the Jiles-Atherton magnetic core model as implemented in SIMetrix. Currently, this is a simplified version to that described by Messrs Jiles and Atherton in their 1986 paper (see ref 1). The basic model is compatible with the version implemented in PSpice but SIMetrix also offers the more complex anhysteric function as an option.

The SIMetrix model is slightly modified from the original in order to overcome non-physical behaviour at flux reversal. The modification is that proposed by Lederer (see ref 2).

This note presents the equations used and their method of implementation but does not attempt to explain the theory behind the model.

THE ORIGINAL MODEL

There are three components to the original Jiles-Atherton model. These are

1. The anhysteric equation. This is the non-linear relationship between magnetising force and flux density without hysteresis.
2. The irreversible magnetisation equation
3. The reversible magnetisation equation.

THE ANHYSTERIC EQUATION

The original equation proposed by Jiles-Atherton is:

$$M_{an} = M_s\left(\coth\left(\left(\frac{H_e}{a}\right) - \frac{a}{H_e}\right)\right) \qquad (1)$$

a and M_s are model parameters. H_e is:

$$H_e = H + \alpha \cdot M$$

where M is the total magnetisation and is the final value we are trying to derive. α is a model parameter. (The addition of the α parameter complicates the implementation and is omitted in the Level 1 version of the model)

M_{an} is the anhysteric core magnetisation. In general magnetisation is related to flux density by the relation:

$$B = (\mu_0 \cdot M)$$

where μ_0 is the permeability of free space equal to $4 \cdot \pi \cdot 10^{-7}$

The Irreversible Magnetisation Equation

$$\frac{dM_{irr}}{dH} = \frac{M_{an} - M_{irr}}{K - \alpha(M_{an} - M_{irr})} \quad (2)$$

This equation must be solved to obtain M_{irr}. K and α are model parameters. The method by which the solution to this equation is obtained in the SIMetrix model implementation is described in the Implementation section below.

The Reversible Magnetisation Equation

$$M_{rev} = C \cdot (M_{an} - M) \quad (3)$$

M is the total magnetisation used to calculate flux density and ultimately the voltage across the winding. It is equal to the sum of the reversible and irreversible components:

$$M = M_{rev} + M_{irr} \quad (4)$$

Final Equations

Combining 3 and 4 above to eliminate M_{rev}:

$$M = \frac{M_{irr} + M_{an} \cdot C}{1 + C} \quad (5)$$

M_{an} is derived from (1):

$$M_{an} = M_s\left(\coth\left(\left(\frac{H_e}{a}\right) - \frac{a}{H_e}\right)\right)$$

M_{irr} is derived from (2):

$$\frac{dM_{irr}}{dH} = \frac{M_{an} - M_{irr}}{K - \alpha(M_{an} - M_{irr})}$$

Implementation

The main difficulty is implementing the solution of equation (2). The simulator already has the ability to solve differential equations with respect to time, so the method chosen is to convert equation (2) to a differential equation in time rather than *H*. The main difficulty with this approach is that it can only work in a time domain simulation. This means that the model will not behave correctly in a DC simulation, but inductors do not take part in DC simulation anyway so this is not an issue in practical applications.

To convert (2) to the time domain we multiply by $\frac{dH}{dt}$

$$\frac{dM_{irr}}{dH} \cdot \frac{dH}{dt} = \frac{M_{an} - M_{irr}}{K - \alpha(M_{an} - M_{irr})} \cdot \frac{dH}{dt}$$

which becomes:

$$\frac{dM_{irr}}{dt} = \frac{M_{an} - M_{irr}}{K - \alpha(M_{an} - M_{irr})} \cdot \frac{dH}{dt}$$

The above describes the full Jiles-Atherton model. In the Level 1 SIMetrix (and also PSpice) version of the model, the α parameter is not included and is assumed to be zero. This simplifies the implementation.

Alternative Anhysteric Equation

PSpice uses a simpler anhysteric equation as follows:

$$M_{an} = M_s \cdot \frac{H}{|H| + A} \qquad (6)$$

SIMetrix uses this version when the AHMODE parameter is set to 0 (the default value).

Air Gap

To model air gap, it is necessary to solve a magnetic equation as follows:

$$H_{core} \cdot L_e + H_{gap} \cdot L_{gap} = i \cdot N$$

We can assume that the flux density in the gap is the same as in the core so:

$$B_{gap} = B_{core}$$

and

$$H_{gap} = \frac{B_{gap}}{\mu_0} = \frac{B_{core}}{\mu_0} = M$$

therefore:

$$H_{core} \cdot L_e + M \cdot L_{gap} = i \cdot N \qquad (7)$$

Calculating Air Gap from UE

The SIMetrix Jiles-Atherton model provides the facility for the user to define a value of effective permeability (Ue). This is implemented by calculating an appropriate air gap. The following relations are used to derive the formula:

$$M = H \cdot \mu_e \qquad (8)$$

$$M = H_{core} \cdot \mu_i \qquad (9)$$

where

$$\mu_i = \frac{C}{(C+1)} \cdot \frac{M_s}{3 \cdot A}$$ for original J-A anhysteric (1)

or

$$\mu_i = \frac{C}{(C+1)} \cdot \frac{M_s}{A}$$ for simplified PSpice version (6)

We can combine eqns 7, 8 and 9 to obtain:

$$H\frac{\mu_e}{\mu_i} + H\mu_e \cdot \frac{L_{gap}}{L_e} = H$$

dividing by H and solving for L_{gap} we obtain:

$$L_{gap} = \frac{\mu_i - \mu_e}{\mu_i \cdot \mu_e}$$

REFERENCES

1. Theory of Ferrmagnetic Hysteresis, DC.Jiles, D.L. Atherton, Journal of Magnetism and Magnetic Materials, 1986 p48-60.
2. On the Parameter Identification and Application of the Jiles-Atherthon Hysteresis Model for Numerical Modelling of Measured Characteristics, D Lederer, H Igarashi, A Kost and T Honma, IEEE Transactions on Magnetics, Vol. 35, No. 3, May 1999